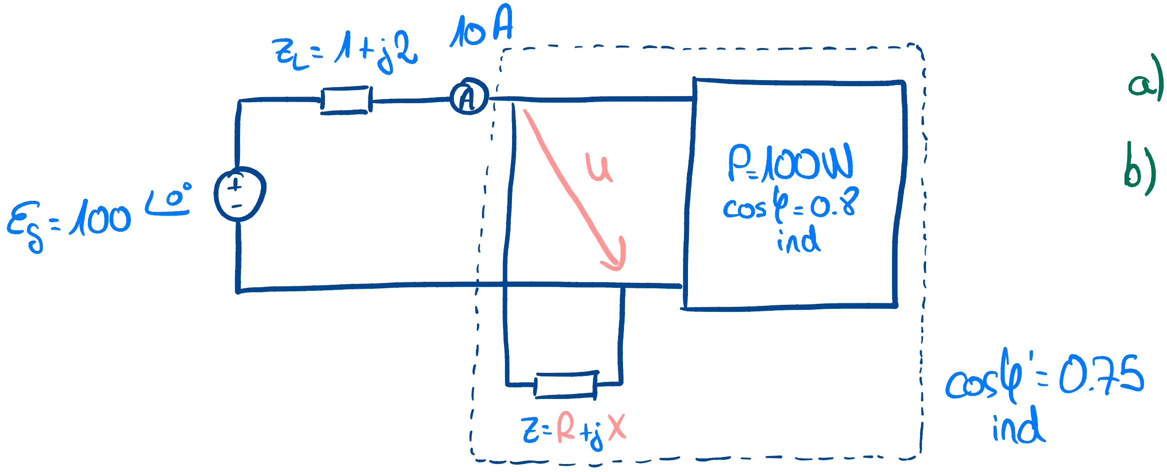
Problema 1



a) U

b) R, X

a) U

Potencia aparente fuente

$$S_g = \mathcal{E}_g \cdot I = 100 \cdot 10 = 1000 \text{ VA}$$

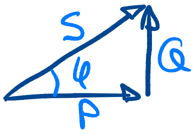
Potencia activa y reactiva línea

$$P_\ell = R_\ell \cdot I^2 = 1 \cdot 10^2 = 1 \cdot 100 = 100 \text{ W}$$
$$Q_\ell = X_\ell \cdot I^2 = 2 \cdot 10^2 = 2 \cdot 100 = 200 \text{ VAr}$$

En el conjunto $\cos\varphi' = 0.75$, inductivo

$$\varphi' = \arccos 0.75 = 41.41°$$

Potencia activa y reactiva conjunto



$$\left.\begin{array}{l} P = S\cos\varphi \rightarrow S = \dfrac{P}{\cos\varphi} \\ Q = S \operatorname{sen}\varphi \end{array}\right\} Q = P\,\frac{\operatorname{sen}\varphi}{\cos\varphi} = P\operatorname{tg}\varphi$$

$$\frac{Q}{P} = \operatorname{tg}\varphi \rightarrow Q = P\operatorname{tg}\varphi$$

$$Q' = P'\operatorname{tg}\varphi' = P'\operatorname{tg}(\arccos 0.75) = P'\operatorname{tg} 41.41 = 0.8819\, P'$$

Entonces, tenemos

$$\left.\begin{array}{l} S_g = \sqrt{(P_e+P')^2+(Q_e+Q')^2} \\ Q' = P'\beta \end{array}\right\} S_g = \sqrt{(P_e+P')^2+(Q_e+P'\beta)^2} \rightarrow$$

$$\rightarrow S_g^2 = (P_e+P')^2 + (Q_e + P'\beta)^2 \rightarrow S_g^2 = P_e^2 + 2P_eP' + P'^2 + Q_e^2 + 2Q_eP'\beta + (P'\beta)^2 \rightarrow$$

$$\rightarrow S_g^2 = P_e^2 + Q_e^2 + P'^2 + 2P_eP' + 2Q_eP'\beta + P'^2\beta^2 \rightarrow$$

$$\rightarrow P'^2(1+\beta^2) + 2P'(P_e+Q_e\beta) - S_g^2 + P_e^2 + Q_e^2 = 0$$

$$P' = \frac{-\cancel{2}(P_e+Q_e\beta) \pm \sqrt{(\cancel{2}(P_e+Q_e\beta))^2 - \cancel{4}(1+\beta^2)(-S_g^2+P_e^2+Q_e^2)}}{\cancel{2}(1+\beta^2)}$$

$$-(P_e+Q_e\beta) = -(100+200\cdot 0.8819) = -276.38$$

$$(P_e+Q_e\beta)^2 = (100+200\cdot 0.8819)^2 = 76385.9044$$

$$(1+\beta^2)(-S_g^2+P_e^2+Q_e^2) = (1+0.8819^2)(-1000^2+100^2+200^2) = -1688860.23$$

$$(1+\beta^2) = 1+0.8819^2 = 1.7777$$

$$P' = \frac{-276.38 \pm \sqrt{76385.9044+1688860.23}}{1.7777} = \begin{array}{l} \xrightarrow{\oplus} P' = 591.91 \text{ W} \leftarrow \text{inductivo} \\ \xrightarrow{\ominus} P' = -902.86 \text{ W} \end{array}$$

Entonces, la potencia reactiva

$$Q' = 0.8819P' = 0.8819\cdot 591.91 = 522.01 \text{ VAr}$$

$$Q = 0.8819P' = 0.8819 \cdot 591.91 = 522.01 \text{ VAr}$$

Considerando I como referencia entre fases

$$I = 10\angle 0^\circ$$

La potencia compleja del conjunto resulta

$$S' = UI^* \to U = \frac{S'}{I^*} = \frac{P' + jQ'}{I^*}$$

$$U = \frac{591.91 + j522.01}{10\angle 0^\circ} = \frac{789.21\angle 41.41^\circ}{10} = 78.921\angle 41.41^\circ$$

Entonces, la fuente de tensión es

$$E_g = IZ + U = 10\angle 0^\circ (1+j2) + 78.921\angle 41.41^\circ =$$

$$= 10(1+j2) + 59.19 + j52.20 = 10 + j20 + 59.19 + j52.20$$

$$= 69.19 + j72.20 = 100\angle 46.22^\circ$$

Según el enunciado, la referencia es E_g, entonces

$$E_g = 100\angle 0^\circ \quad I = 10\angle -46.22^\circ \quad U = 78.921\angle -4.81^\circ$$

b) R, X

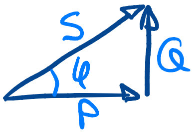
Haciendo balance de potencias activas

$$P' = P_Z + P \to P_Z = P' - P = 591.91 - 100 = 491.91 \text{ W}$$

El $\cos\varphi = 0.8$, inductivo

$$\varphi = \arccos 0.8 = 36.87^\circ$$

Potencia activa y reactiva



$$\left.\begin{array}{l} P = S\cos\varphi \to S = \dfrac{P}{\cos\varphi} \\ Q = S\,\operatorname{sen}\varphi \end{array}\right\} Q = P\,\frac{\operatorname{sen}\varphi}{\cos\varphi} = P\operatorname{tg}\varphi$$

$$\frac{Q}{P} = \operatorname{tg}\varphi \to Q = P\operatorname{tg}\varphi$$

$$Q = P\operatorname{tg}\varphi = 100\operatorname{tg}(\arccos 0.8) = 75 \text{ VAr}$$

Haciendo balance de potencias reactivas

$$Q' = Q_z + Q \rightarrow Q_z = Q' - Q = 522.01 - 75 = 447.01 \text{ VAr}$$

Entonces

$$S_z = y^* U^2 \rightarrow P_z + jQ_z = y^* U^2 \rightarrow y^* = \frac{P_z + jQ_z}{U^2}$$

$$y^* = \frac{491.91 + j\,447.01}{78.921^2} = \frac{664.675 \angle 42.26°}{6228.52} = 0.1067 \angle 42.26°$$

$$y = 0.1067 \angle -42.26°$$

Entonces

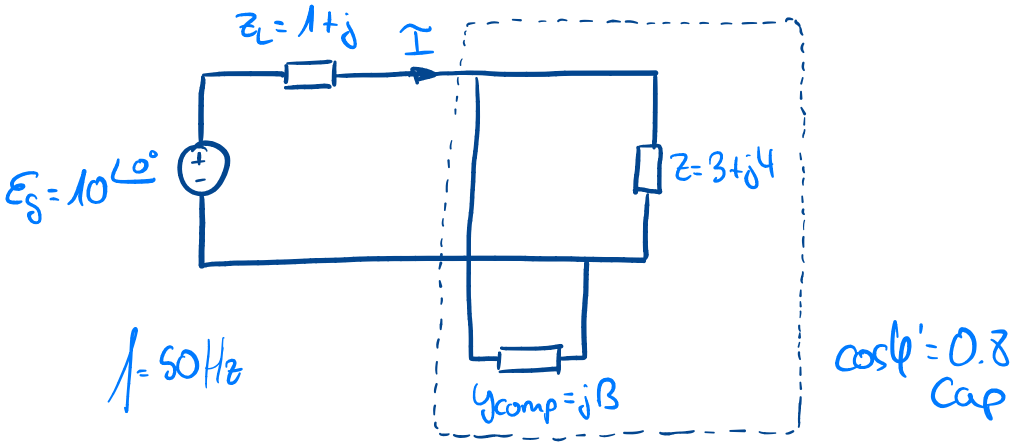
$$Z = \frac{1}{y} = \frac{1}{0.1067 \angle -42.26°} = 9.3721 \angle 42.26° = 6.936 + j6.303$$

Entonces

$$R = 6.936\ \Omega$$

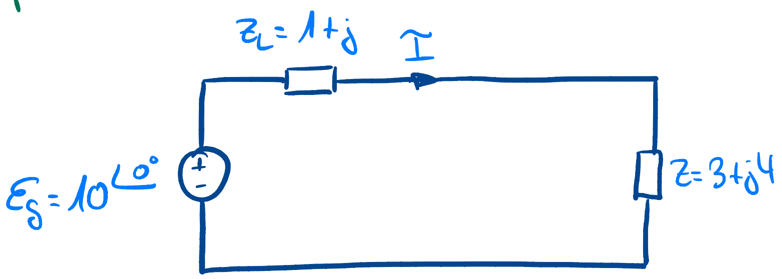
$$X = 6.303\ \Omega$$

Problema 2



a) pérdidas línea
b) elemento compensación
c) pérdidas línea
d) fasor Intensidad

a) pérdidas línea

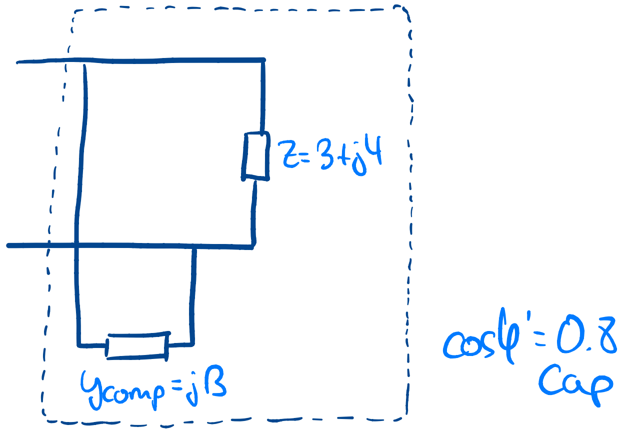


$$\mathcal{E}_g = I(Z_L+Z) \Rightarrow I = \frac{\mathcal{E}_g}{Z_L+Z} = \frac{10\angle 0°}{1+j+3+j4} = \frac{10}{4+j5} =$$

$$= \frac{10}{6.40\angle 51.34°} = 1.5625\angle -51.34°$$

$$P_L = R_L \cdot I^2 = 1 \cdot 1.5625^2 = 2.4414\ W$$

b) Elemento compensación



La admitancia del conjunto es

$$Y = Y_{comp} + \frac{1}{Z} = jB + \frac{1}{3+j4} = jB + \frac{3-j4}{9+16} = jB + \frac{3-j4}{25} = \frac{j25B+3-j4}{25}$$

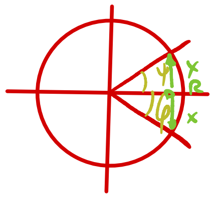
$$Y = \frac{3}{25} + \frac{j}{25}(25B-4)$$

En el conjunto, $\cos\varphi' = 0.8$ capacitivo

$$\varphi' = -\text{acos}\,\varphi' = -\text{acos}\,0.8 = -36.87°$$

El ángulo correspondiente a la admitancia es

$$\psi = -\varphi' = 36.87°$$



$$\text{tg}\,\psi = \frac{X}{R} = -\text{tg}\,\varphi = \frac{-X}{R}$$

Entonces

$$\text{tg}\,\psi = \frac{X}{R} = \frac{\frac{1}{\cancel{25}}(25B-4)}{\frac{3}{\cancel{25}}} \rightarrow 3\,\text{tg}\,\psi = 25B-4 \rightarrow B = \frac{3\,\text{tg}\,\psi+4}{25}$$

$$B = \frac{3\,\text{tg}\,36.87+4}{25} = 0.25$$

$$\left.\begin{aligned} Z_L &= j\omega L \\ Z_C &= \frac{1}{j\omega C}\end{aligned}\right\} \quad B > 0 \rightarrow \text{condensador}$$

$$Z_C = \frac{1}{j\omega C} \rightarrow \frac{1}{jB} = \frac{1}{j\omega C} \rightarrow C = \frac{B}{\omega} = \frac{B}{2\pi f} = \frac{0.25}{2\pi\cdot 50} =$$

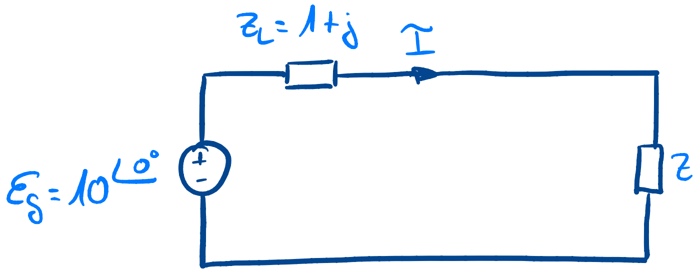
$$= 0.796\,\text{mF}$$

impedancia

$$Z = \frac{1}{Y} = \frac{1}{\frac{3}{25}+\frac{j}{25}(25B-4)} = \frac{25}{3+j(25B-4)} = \frac{25(3-j(25B-4))}{9+(25B-4)^2}$$

$$\text{tg}\,\varphi = \frac{X}{R} = \frac{\frac{\cancel{25}(-(25B-4))}{\cancel{9+(25B-4)^2}}}{\frac{\cancel{25}\cdot 3}{\cancel{9+(25B-4)^2}}} = \frac{-(25B-4)}{3} \neq \text{tg}\,\psi = \frac{25B-4}{3}$$

c) pérdidas línea



Resistencias en paralelo

$$\left.\begin{array}{l} y = \frac{3}{25} + \frac{j}{25}(25B-4) \\ B = 1/4 \end{array}\right\} \; y = \frac{3}{25} + \frac{j}{25}\left(\frac{25}{4} - 4\right) = \frac{3}{25} + \frac{j}{25}\frac{9}{4} = 0.15\angle 36.87°$$

$$z = \frac{1}{y} = \frac{1}{0.15\angle 36.87°} = 6.67\angle -36.87° = 5.34 - j4$$

De esta manera, tenemos

$$E_g = I(Z_L + Z) \rightarrow I = \frac{E_g}{Z_L + Z} = \frac{10\angle 0°}{1+j+5.34-j4} = \frac{10}{6.34-j3} =$$

$$= \frac{10}{7.01\angle -25.32°} = 1.43\angle 25.32°$$

As perdas na línea resultan

$$P_L = R_L I^2 = 1 \cdot 1.43^2 = 2.0449\ W$$

d) fasor intensidad

$$I = 1.43\angle 25.32°$$

Resumen

Potencia aparente

$S = E_g \cdot I \equiv S = U \cdot I \equiv S = y^* U^2$ VA

Potencia activa (pérdidas) y reactiva

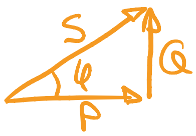
$P = R \cdot I^2$ W

$Q = X \cdot I^2$ VAr

Cos φ

- inductivo → $\varphi = a\cos\varphi$
- capacitivo → $\varphi = -a\cos\varphi$

Potencia activa y reactiva

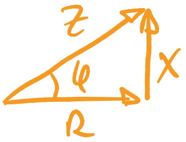


$$\left.\begin{array}{l} P = S\cos\varphi \rightarrow S = \dfrac{P}{\cos\varphi} \\ Q = S\,\mathrm{sen}\varphi \end{array}\right] Q = P\,\frac{\mathrm{sen}\varphi}{\cos\varphi} = P\,\mathrm{tg}\varphi$$

$$\frac{Q}{P} = \mathrm{tg}\varphi \rightarrow Q = P\,\mathrm{tg}\varphi$$

$$S = \sqrt{P^2 + Q^2}$$

Resistencia y reactancia



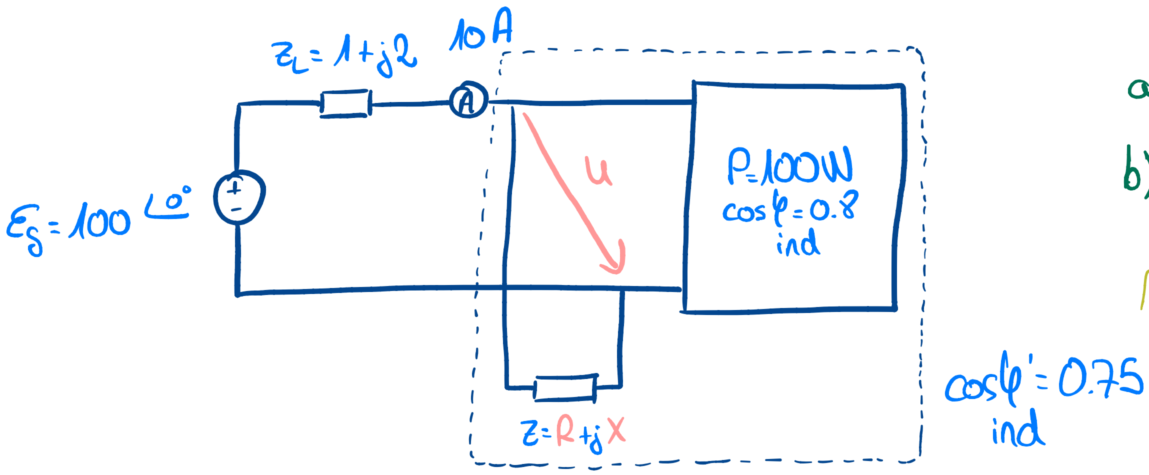
$$\left.\begin{array}{l} R = Z\cos\varphi \rightarrow R = \dfrac{Z}{\cos\varphi} \\ X = Z\,\mathrm{sen}\varphi \end{array}\right] X = R\,\frac{\mathrm{sen}\varphi}{\cos\varphi} = R\,\mathrm{tg}\varphi$$

$$\frac{X}{R} = \mathrm{tg}\varphi \rightarrow X = R\,\mathrm{tg}\varphi$$

$$Z = \sqrt{R^2 + X^2}$$

Problema 1

impedancia



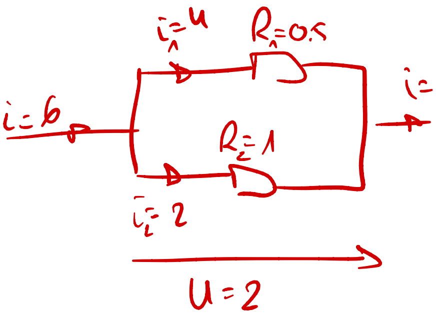
a) U

b) R, X ← reactancia
↑
resistencia

a) U

Necesitamos $U^{\angle \alpha_u}$?

- $\mathcal{E}_g^{\angle \alpha_\mathcal{E}} = Z_L I^{\angle \alpha_I} + U^{\angle \alpha_u}$
- $S_u^{\angle \alpha_u} = U^{\angle \alpha_u} I^{\angle \alpha}$?
- $S_u = \sqrt{P_u^2 + Q_u^2}$?



$P_1 = U i_1 = 2 \cdot 4 = 8$
$P_2 = U i_2 = 2 \cdot 2 = 4$ } 12

$P = U i = 2 \cdot 6 = 12$

- $S_g = \sqrt{P_g^2 + Q_g^2} = \sqrt{(P_{ZL} + P_u)^2 + (Q_{ZL} + Q_u)^2}$
- $P_u = S_u \cos(+\text{acos}\,\varphi)$
  $Q_u = S_u \,\text{sen}(+\text{acos}\,\varphi)$ } $\frac{Q_u}{P_u} = \frac{\cancel{S_u}\,\text{sen}(+\text{acos}\,\varphi)}{\cancel{S_u}\cos(+\text{acos}\,\varphi)} = tg(+\text{acos}\,\varphi) \to$

$\to Q_u = P_u \, tg(+\text{acos}\,\varphi)$

$$S_g = \sqrt{(P_{ZL} + P_u)^2 + (Q_{ZL} + P_u tg(+\text{acos}\,\varphi))^2}$$

$S_g = \sqrt{P_g^2 + Q_g^2} = \mathcal{E}_g I = 1000\ VA$
$P_{ZL} = R I^2 = 1 \cdot 10^2 = 100\ W$
$Q_{ZL} = X I^2 = 2 \cdot 10^2 = 200\ VAr$
$tg(+\text{acos}\,\varphi) = 0.8819$

} ⇒

$$\Rightarrow P_u = \frac{-276.38 + \sqrt{76385.9044 + 1688860.23}}{1.7777} = 591.91\ W$$

Potencia activa → tiene que ser positiva, ∄ elementos que consuman potencia activa

Undo

$Q_u = P_u\, tg(+acos(\varphi)) = 522.01\,VAr$

$S_u^{\angle\alpha_u} = P_u + jQ_u$

$S_u^{\angle\alpha_u} = U^{\angle\alpha_u} I^{\angle\alpha_I *} \rightarrow U^{\angle\alpha_u} = \frac{S_u^{\angle\alpha_u}}{I^{\angle\alpha_I *}} = 78.921^{\angle 41.41^\circ}\ V$

? → considero 0°

$\mathcal{E}_g^{\angle\alpha_\mathcal{E}} = Z\, I^{\angle\alpha_I} + U^{\angle\alpha_u} = 100^{\angle 46.22^\circ}\ V$ debería ser 0° (enunciado)

$U^{\angle\alpha_u} = 78.921^{\angle 41.41 - 46.22} = 78.921^{\angle -4.81^\circ}\ V$

b) R, X

$P_z = I^2 R = U^2/R$?
$Q_z = I^2 X = U^2/X$?

- $S^{\angle\alpha_S} = P + jQ = U^2 y^{\angle\alpha_z *} = \overset{?}{I^2}\, Z^{\angle\alpha_z}$
- $P_u = P_z + P \rightarrow P_z = P_u - P = 591.91 - 100 = 491.91\,W$
- $Q_u = Q_z + Q \rightarrow Q_z = Q_u - Q = 522.01 - 100\,tg(+acos\,0.8) = 447.01\,VAr$

$\rightarrow 491.91 + j447.01 = 78.921^2\, y^{\angle\alpha_z *}$

$y^{\angle\alpha_z *} = \frac{664.68^{\angle 42.26^\circ}}{6228.52} \rightarrow y^{\angle\alpha_z *} = 0.1067^{\angle 42.26^\circ} = 0.079 + j0.072\ 1/\Omega$

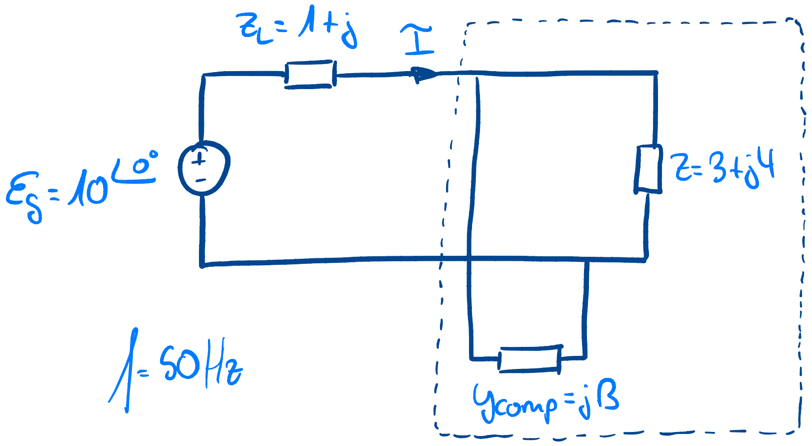
$y^{\angle\alpha_z} = 0.079 - j0.072 = 0.1069^{\angle -42.35}\ 1/\Omega$

$Z^{\angle\alpha_z} = \frac{1}{y^{\angle\alpha_z}} = \frac{1}{0.1069^{\angle -42.35}} = 9.35^{\angle 42.35} = 6.91 + j6.30\ \Omega$

¿bobina o condensador?

$R = 6.91\ \Omega$
$X = 6.30\ \Omega$

Problema 2

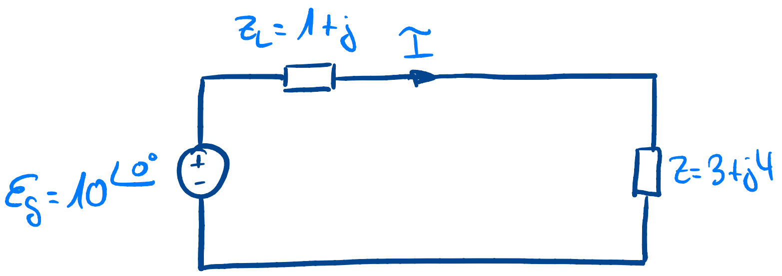


$f = 50\,Hz$

$\cos\varphi' = 0.8$ Cap

a) pérdidas línea
b) elemento compensación
c) pérdidas línea
d) fasor Intensidad

a) pérdidas línea



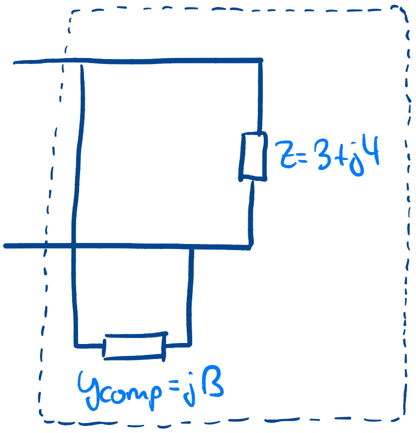
$$P_L = I^2 R_L$$

$$\mathcal{E}_g^{\angle} = I^{\angle}(Z_L + Z) \rightarrow I^{\angle} = \frac{\mathcal{E}_g}{Z_L + Z} = \frac{10^{\angle 0}}{1+j+3+j4} = \frac{10}{4+j5} =$$

$$= \frac{10}{6.40\angle 51.34} = 1.5625\angle -51.34°$$

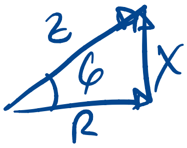
$$P_L = 1.5625^2 \cdot 1 = 2.44 \text{ W}$$

b) elemento compensación



$\cos\varphi' = 0.8$
Cap

$P^{(\alpha} = R^{(\alpha} I^2$
$Q^{(\alpha} = X^{(\alpha} I^2$



$$\left.\begin{array}{l} R = Z\cos\varphi \\ X = Z\,\text{sen}\,\varphi \end{array}\right| \frac{X}{R} = \frac{\cancel{Z}}{\cancel{Z}}\frac{\text{sen}\,\varphi}{\cos\varphi} = tg\,\varphi \rightarrow X = R\,tg(-a\cos(\varphi))$$

ⓐ

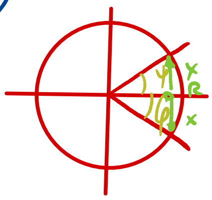
$$y = y_{comp} + \frac{1}{Z} = jB + \frac{1}{3+j4} = jB + \frac{3-j4}{9+16} = jB + \frac{3-j4}{25} = \frac{3}{25} + j\left(B - \frac{4}{25}\right)$$

$$Z = \frac{1}{y} = \frac{1}{\frac{3}{25} + j\left(B - \frac{4}{25}\right)} = \frac{25}{3+j(25B-4)} = \frac{25(3-j(25B-4))}{9+(25B-4)^2}$$

$$\frac{\cancel{25}(-(25B-4))}{9+\cancel{(25B-4)^2}} = \frac{\cancel{25}\cdot 3}{\cancel{9+(25B-4)^2}}\,tg\,\varphi \rightarrow 3\,tg\,\varphi = -25B+4 \rightarrow$$

$$\rightarrow B = \frac{-3\,tg\,\varphi + 4}{25} = 0.25$$

ⓑ



$$tg\,\psi = \frac{X}{R} = -tg\,\varphi = \frac{-X}{R}$$

$Z = Z\angle\varphi$
$y = y\angle\psi$

$$y\angle\psi = \frac{1}{Z\angle\varphi} \left\langle \begin{array}{l} y = \frac{1}{Z} \\ \psi = -\varphi \end{array}\right.$$

$$tg\,\psi = \frac{X}{R} \rightarrow -tg\,\varphi = \frac{X}{R} \rightarrow -tg\,\varphi = \frac{25B-4/\cancel{25}}{3/\cancel{25}} \rightarrow -3\,tg\,\varphi = 25B-4 \rightarrow B = \frac{-3\,tg\,\varphi+4}{25} = 0.25$$

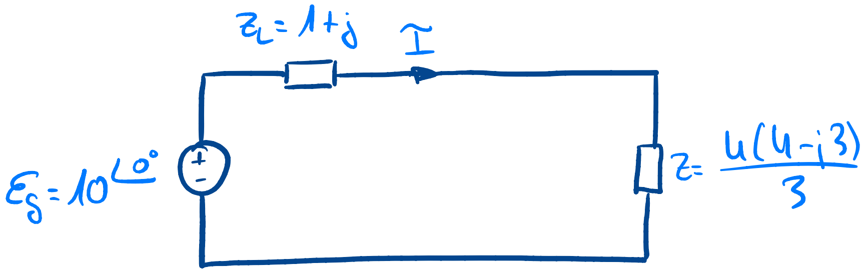
$B=0.25 \rightarrow y_{comp} = j0.25 \rightarrow z_{comp} = \frac{1}{y_{comp}} = \frac{1}{j0.25} = -j4$

$\left.\begin{array}{l} Z_C = \frac{1}{jC\omega} \\ Z_L = jL\omega \end{array}\right\} Z<0 \rightarrow$ condensador $\quad -4 = \frac{-1}{C\cdot 2\pi \cdot 50} \rightarrow C = 0.796\,mF$

c) pérdidas línea

$$\left.\begin{array}{l} Z = \frac{25(3-j(25B-4))}{9+(25B-4)^2} \\ B=0.25 \end{array}\right\} Z = \frac{25(3-j(25\cdot 0.25-4))}{9+(25\cdot 0.25-4)^2} = \frac{25(3-j\,9/4)}{9+81/16} =$$

$$= \frac{25/4\,(12-j9)}{(144+81)/16} = \frac{4\cdot 25(12-j9)}{225} = \frac{4(12-j9)}{9} = \frac{4(4-j3)}{3}$$



$$\mathcal{E}_g^{\angle \alpha_g} = I^{\angle \alpha}(Z_L+Z) \rightarrow I^{\angle \alpha} = \frac{\mathcal{E}_g^{\angle \alpha_g}}{Z_L+Z} = \frac{10^{\angle 0°}}{1+j+\frac{4(4-j3)}{3})} =$$

$$= \frac{10}{19/3 - j\,9/3} = \frac{30}{19-j9} = \frac{30(19-j9)}{361+81} = \frac{30(19-j9)}{442} = \frac{15(19-j9)}{221} =$$

$$= 1.427 \angle 25.346°$$

Pérdidas:

$P = RI^2 = 1\cdot 1.427^2 = 2.036\ W$

d) fasor intensidad

$I^{\angle \alpha} = 1.427 \angle 25.346°$